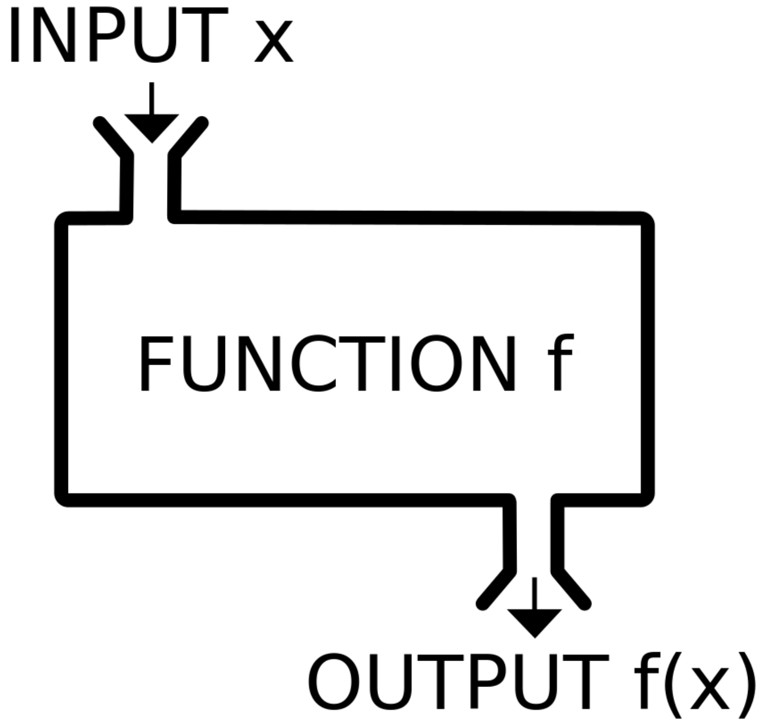
Truth Tables

Digital Electronics

Wolfgang Neff

Truth Tables (1)

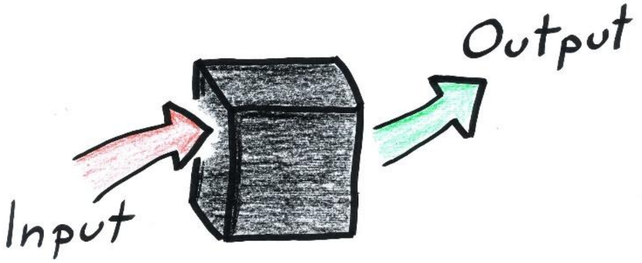
- Functions
  - Process input
  - Produce output
  - They have
    - Parameters
      - Input
  - They return
    - Results
      - Output



Truth Tables (2)

- Logical Functions
  - Are expression of
    - Parameters (a, b, c, …)
    - Operators (¬, ∧, ∨, …)
    - Functions (φ, ψ, χ, …)
  - Alternative terms
    - Logic function
    - Boolean function
    - Switching function

$$\underbrace{\phi(a,b)}_{\text{Parameters}} = \underbrace{a \wedge b}_{\text{Expression}}$$



0, 1	φ(a,b)	0
Arguments	Function	Result

Truth Tables (3)

- Logical Functions (continued)
  - Arithmetic and truth functions are different
    - Arithmetic functions are infinite
    - Truth functions are finite

$f(1,1) = 2$
$f(1,2) = 3$
$f(1,3) = 4$
$f(1,4) = 5$
…
Arguments never end

Arithmetic function $f(x,y) = x+y$

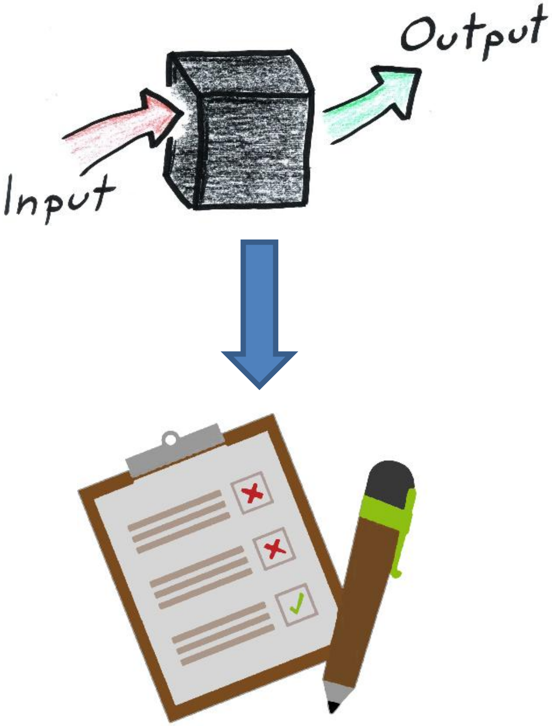
$\varphi(0,0) = 0$
$\varphi(0,1) = 0$
$\varphi(1,0) = 0$
$\varphi(1,1) = 1$

No arguments any more

Truth function $\varphi(a,b) = a \wedge b$

Truth Tables (4)

- Logical Functions (continued)
  - A truth table can be made
    - All possible inputs (arguments)
    - All possible outputs (results)
  - Alternative terms
    - Truth table
    - Switching table
    - State table



Truth Tables (5)

- Logical Functions (finished)
  - Example
    - $\phi(a,b) = a \land b$
  - Description
    - Two parameters
      - a, b
    - Four arguments
      - (0,0), (0,1), (1,0), (1,1)
    - One result
      - 0 or 1

a	b	$\phi(a,b)$
0	0	0
0	1	0
1	0	0
1	1	1

Truth Tables (6)

- Examples
  - Functions with one parameter: ϕ(a)
  - Functions with two parameters: ϕ(a,b)

a	ϕ(a)
0	…
1	…

a	b	ϕ(a,b)
0	0	…
0	1	…
1	0	…
1	1	…

Truth Tables (7)

- Construction
  - Number of columns
    - Number of parameters: n
      - Plus auxiliary columns and a column for the result
  - Number of rows
    - Number of arguments: 2^n
  - First column
    - Fifty-fifty: ½ column 0, ½ column 1

Truth Tables (8)

- Construction (continued)
  - Second column
    - Fifty-fifty but twice as fast: *¼ 0, ¼ 1, ¼ 0, ¼ 1*
  - And so on …
  - Checklist
    - First rows starts with: 0 0 0 0 …
    - Last rows ends with: 1 1 1 1 …
    - Last column is alternating*:* 0 1 0 1 0 1 …
    - Sequence of rows: binary natural numbers (0 1 2 3 …)

Truth Tables (9)

- Functions with four parameters: ϕ(a,b,c,d)

a	b	c	d
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1

continued on the right

a	b	c	d
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

Truth Tables (10)

- Logical Equivalence
  - Truth tables are identical
  - ϕ ↔ ψ

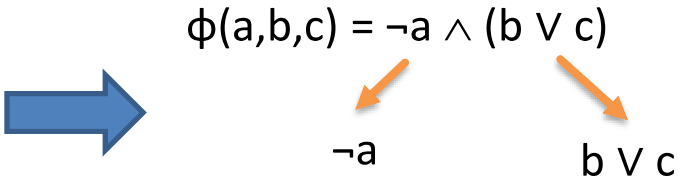
a	b	ϕ(a,b)
0	0	0
0	1	1
1	0	1
1	1	0

a	b	ψ(a,b)
0	0	0
0	1	1
1	0	1
1	1	0

Truth Tables (11)

- Creation (first method)
  - Mathematician's method
  - Term by term
    - Terms: compose the expression of a function
    - Functions: span a tree of terms

$$\phi(a,b,c) = \underbrace{\underbrace{\neg a}_{1^{st}\text{ term}} \wedge \underbrace{(b \vee c)}_{2^{nd}\text{ term}}}_{3^{rd}\text{ term}}$$



$\phi(a,b,c) = \neg a \wedge (b \vee c)$

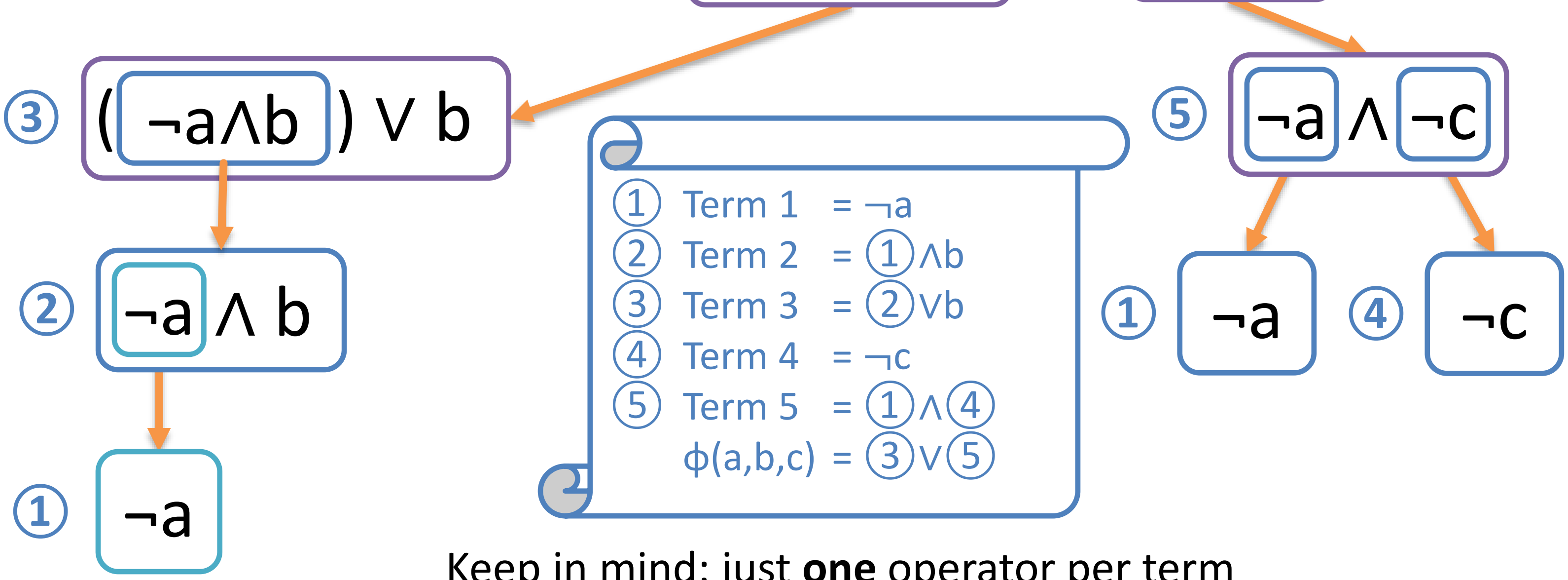
$\neg a$

$b \vee c$

The goal is one operator per term

Truth Tables (12)

- Creation (first method, continued)
  - Example: $\phi(a,b,c) = (\boxed{(\neg a \wedge b) \vee b}) \vee (\boxed{\neg a \wedge \neg c})$



③ $(\boxed{\neg a \wedge b}) \vee b$

② $\boxed{\neg a} \wedge b$

① $\neg a$

⑤ $\boxed{\neg a} \wedge \boxed{\neg c}$

① $\neg a$

④ $\neg c$

① Term 1 = ¬a
② Term 2 = ①∧b
③ Term 3 = ②∨b
④ Term 4 = ¬c
⑤ Term 5 = ①∧④
ϕ(a,b,c) = ③∨⑤

Keep in mind: just **one** operator per term

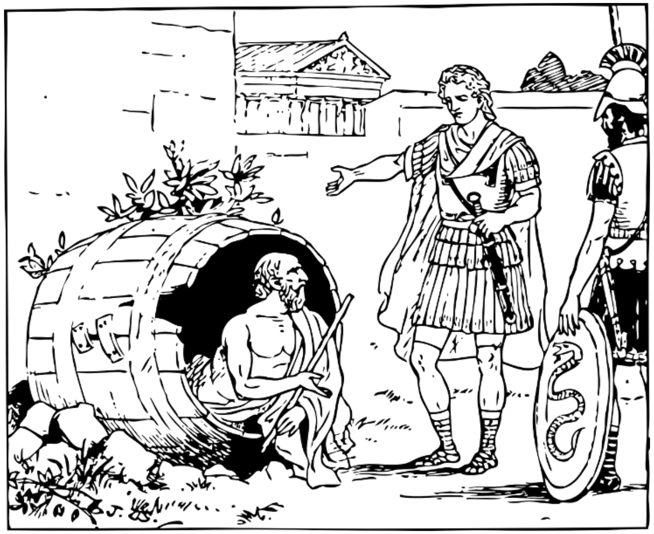
Truth Tables (13)

- Creation (first method, finished)
  - Example: $\phi(a,b,c) = ((\neg a \wedge b) \vee b) \vee (\neg a \wedge \neg c)$

			①	②	③	④	⑤	**ϕ(a,b,c)**
a	b	c	¬a	①∧b	②∨b	¬c	①∧④	③∨⑤
0	0	0	1	0	0	1	1	1
0	0	1	1	0	0	0	0	0
0	1	0	1	1	1	1	1	1
0	1	1	1	1	1	0	0	1
1	0	0	0	0	0	1	0	0
1	0	1	0	0	0	0	0	0
1	1	0	0	0	1	1	0	1
1	1	1	0	0	1	0	0	1

Truth Tables (14)

- Creation (second method)
  - – Philosopher's method
    - Put each symbol in a column
    - Fill in the arguments
    - Perform the operations
      - – Fill the column with the results
      - – The last column shows the result of the truth function.



a	b	a	∧	b
0	0	0	0	0
0	1	0	0	1
1	0	1	0	0
1	1	1	1	1

Truth Tables (15)

- Creation (second method, continued)
  - Example: ϕ(a,b,c) = ((¬a∧b) ∨ b) ∨ (¬a∧¬c)

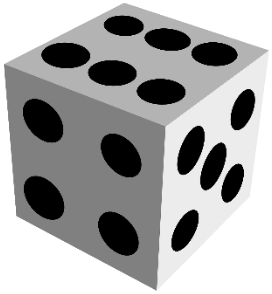
Sequence					2	1	4	3		6	5		12		8	7	11	10	9	
a	b	c	(	(	¬	a	∧	b	)	∨	b	)	∨	(	¬	a	∧	¬	c	)
0	0	0			1	0	0	0		0	0		1		1	0	1	1	0	
0	0	1			1	0	0	0		0	0		0		1	0	0	0	1	
0	1	0			1	0	1	1		1	1		1		1	0	1	1	0	
0	1	1			1	0	1	1		1	1		1		1	0	0	0	1	
1	0	0			0	1	0	0		0	0		0		0	1	0	1	0	
1	0	1			0	1	0	0		0	0		0		0	1	0	0	1	
1	1	0			0	1	0	1		1	1		1		0	1	0	1	0	
1	1	1			0	1	0	1		1	1		1		0	1	0	0	1	

Truth Tables (16)

- Don’t-Care Terms
  - Truth tables can be incomplete
  - Length of truth table is fixed: 2^n
  - Unused rows are called don’t-care terms
  - They must not be omitted
  - They are marked by X

Truth Tables (17)

- Don’t-Care Terms (continued)
  - Example: Which face of a dice has six pips?



n	a	b	c	ϕ(a,b,c)	Face
0	0	0	0	X	invalid
1	0	0	1	0	⚀
2	0	1	0	0	⚁
3	0	1	1	0	⚂
4	1	0	0	0	⚃
5	1	0	1	0	⚄
6	1	1	0	1	⚅
7	1	1	1	X	invalid

Truth Tables (18)

- Conclusion
  - Truth tables describe logic functions
    - One function has exactly one truth table
  - Truth tables are not unique
    - Many functions have the same truth table
    - These functions are logically equivalent
  - Some of these functions can easily be found
    - Truth table → logic function -> digital circuit