

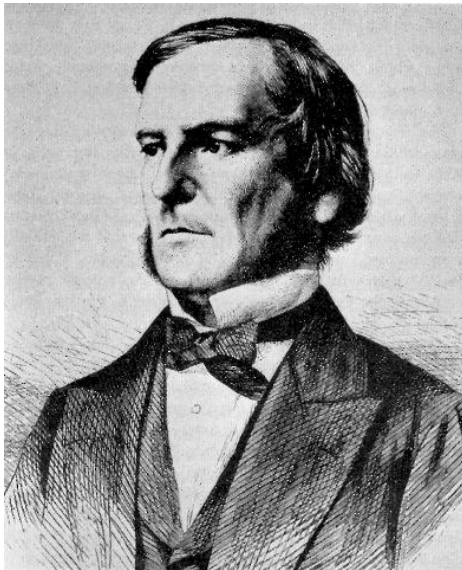
# Logical Operators

Digital Electronics

Wolfgang Neff

# Boolean Algebra (1)

- George Boole
  - British mathematician and philosopher
  - Mathematical foundation of computer science



- \* 2 Nov 1815 in England
- † 8 Dec 1864 in Ireland

# Boolean Algebra (2)

- Calculating with Truth

- True  $\rightarrow$  1

- False  $\rightarrow$  0

Something that can be true or false.  
Calculating with truth is difficult.

$$1+1 = 2$$

$$0-1 = -1$$

$$1 \div 0 = \perp$$



A new kind of  
mathematics  
is necessary



# Logical Operators (1)

- Operate on logical values
  - True/False, On/Off, High/Low, 1/0
- Alternative terms
  - Logical operator
    - Logical connective
    - Boolean operator
  - Logical value
    - Truth value
    - Boolean value



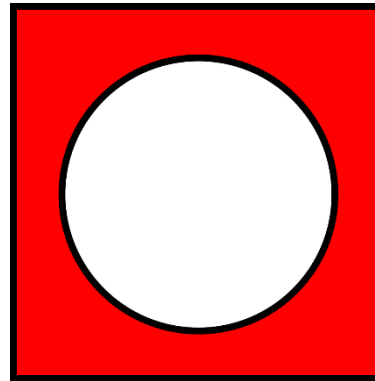
# Logical Operators (2)

- Negation

- Symbol:  $\neg$  (NOT, sometimes  $\neg A \rightarrow \bar{A}$ )
- Meaning: logical not (contrary of ...)
- Definition:

a	$\neg a$
0	1
1	0

Truth Table



Venn diagram

# Logical Operators (3)

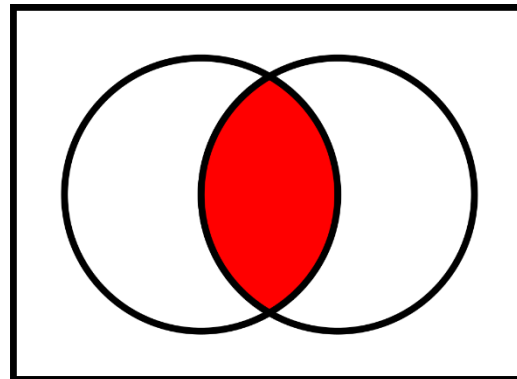
- Conjunction

- Symbol:  $\wedge$  (AND)

- Meaning: logical and (both must be true)

- Definition:

- | a | b | $a \wedge b$ |
|---|---|--------------|
| 0 | 0 | 0            |
| 0 | 1 | 0            |
| 1 | 0 | 0            |
| 1 | 1 | 1            |



# Logical Operators (4)

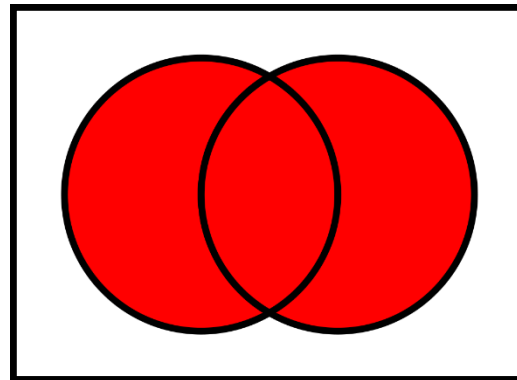
- Disjunction

- Symbol:  $\vee$  (OR)

- Meaning: logical or (at least one must be true)

- Definition:

- | a | b | $a \vee b$ |
|---|---|------------|
| 0 | 0 | 0          |
| 0 | 1 | 1          |
| 1 | 0 | 1          |
| 1 | 1 | 1          |



# Logical Operators (5)

- Antivalence

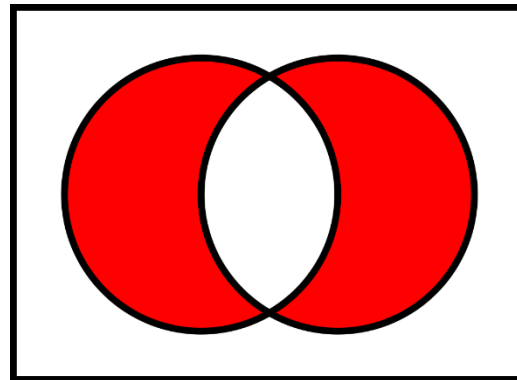
- Symbol:  $\oplus$  (XOR, also  $\Leftrightarrow$  or  $\underline{\vee}$ )

- Meaning: exclusive OR (one but not both must be true)

- Definition:

- | a | b | $a \oplus b$ |
|---|---|--------------|
| 0 | 0 | 0            |
| 0 | 1 | 1            |
| 1 | 0 | 1            |
| 1 | 1 | 0            |

Not equivalent  
(not equal)



# Logical Operators (6)

- Alternative Denial (Sheffer stroke)

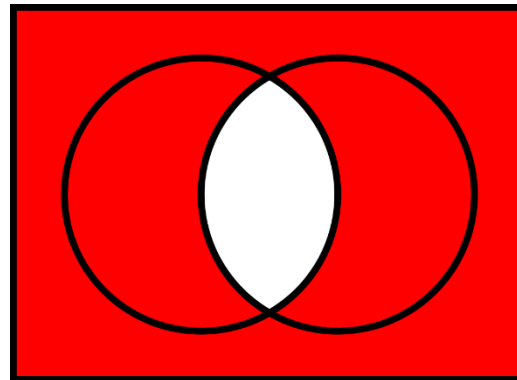
- Symbol:  $|$  (NAND, also  $\uparrow$  or  $\bar{\wedge}$ )

There is no alternative

- Meaning: negation of AND (at least one must be false)

- Definition:

- | a | b | a b |
|---|---|-----|
| 0 | 0 | 1   |
| 0 | 1 | 1   |
| 1 | 0 | 1   |
| 1 | 1 | 0   |



# Logical Operators (7)

- Joint Denial (Peirce arrow)

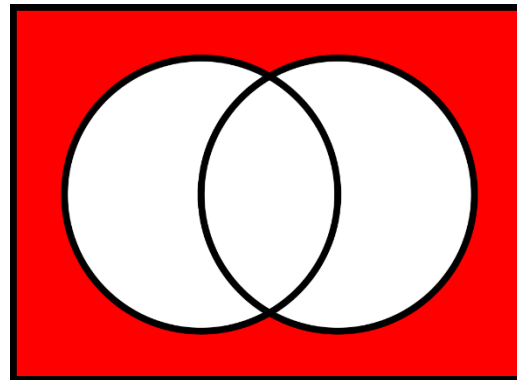
- Symbol:  $\downarrow$  (NOR, also  $\bar{\vee}$ )

Nothing is true

- Meaning: negation of OR (both must be false)

- Definition:

- | a | b | $a \downarrow b$ |
|---|---|------------------|
| 0 | 0 | 1                |
| 0 | 1 | 0                |
| 1 | 0 | 0                |
| 1 | 1 | 0                |



# Laws of Logic (1)

- Double Negation
  - It is not true that it is not true
    - It is true
  - Expressed as a formula
    - $\neg(\neg a) \leftrightarrow a$
  - Proof

a	$\neg a$	$\neg\neg a$
0	1	0
1	0	1



$$a \leftrightarrow \neg\neg a$$

# Laws of Logic (2)

- De Morgan's Law
  - Negation of parentheses
    - Not (A and B) *is equal to* not A or not B
    - Not (A or B) *is equal to* not A and not B
  - Expressed as a formulas
    - $\neg(a \wedge b) \leftrightarrow \neg a \vee \neg b$
    - $\neg(a \vee b) \leftrightarrow \neg a \wedge \neg b$

# Laws of Logic (3)

- De Morgan's Law (continued)

- Example

- All numbers not between 1 and 5

- $\neg(n \geq 1 \wedge n \leq 5) \rightarrow \neg(n \geq 1) \vee \neg(n \leq 5) \rightarrow n < 1 \vee n > 5 \rightarrow \dots, -1, 0, 6, 7, \dots$
- $\neg(n \geq 1 \wedge n \leq 5) \rightarrow n \leq 1 \wedge n \geq 5 \rightarrow \emptyset \rightarrow$  there are no such numbers

Keep in Mind!  
It's Important



# NAND Form

- Complete set of logical operators
  - NOT, AND, OR
  - NAND (or NOR)
    - Just one operator (and easy to realize on silicon)
- Transformation
  - $\neg A \rightarrow A|A$
  - $A \wedge B \rightarrow (A|B)|(A|B)$
  - $A \vee B \rightarrow (A|A)|(B|B)$

