

Capacitors and Inductors

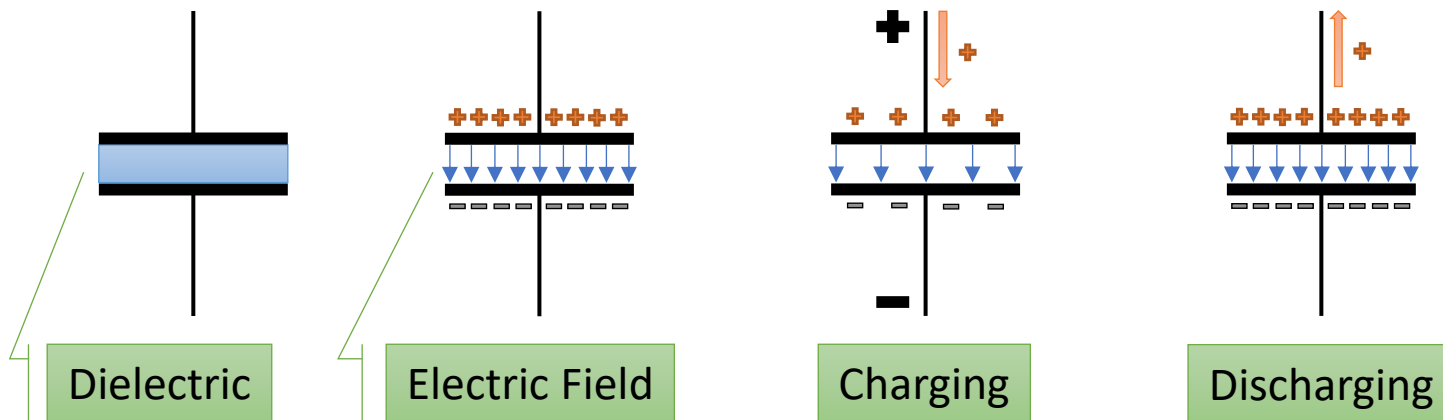
Electrical Engineering

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Capacitors (1)

- Basic Features

- Accumulates electric charges on two surfaces
 - The surfaces are insulated from each other
- Stores electrical energy in an electric field
- It can be charged and discharged



Capacitors (2)

- Charging

- Basic Relations

- $R = \frac{U}{I}$

- $I = \frac{dQ}{dt} = \dot{Q}$

- $Q = C \cdot U$

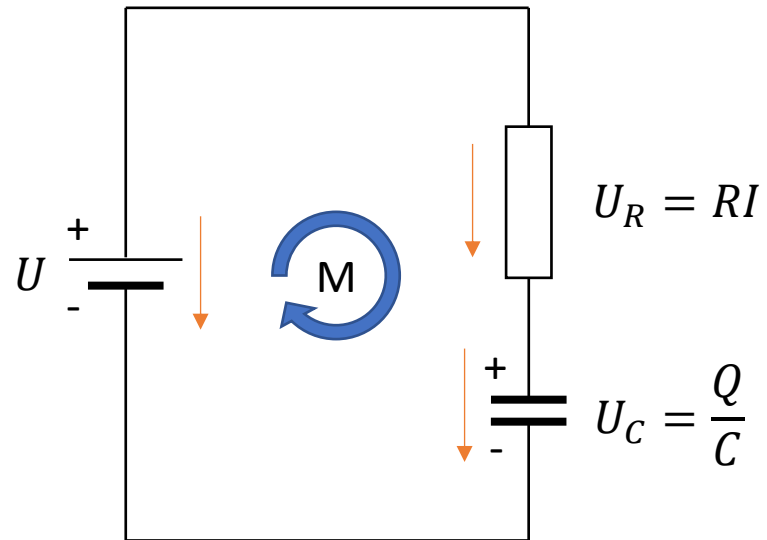
- Kirchhoff's circuit law

- $U_R + U_C - U = 0$

- $R\dot{Q} + \frac{Q}{C} - U = 0$

- Differential Equation

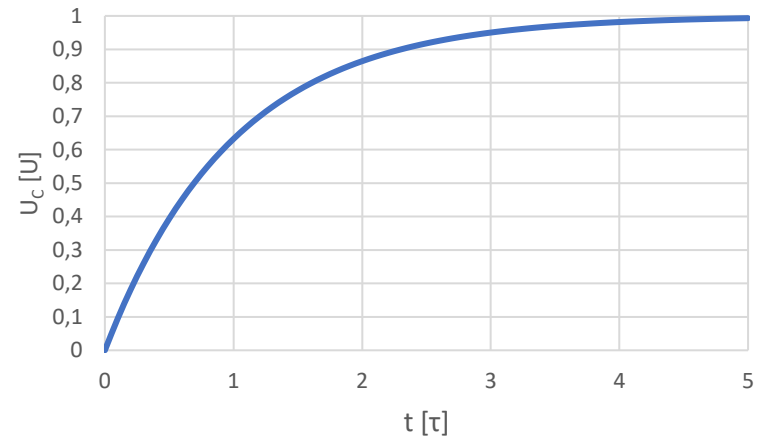
- $\dot{Q} = \frac{1}{R}U - \frac{1}{RC}Q$



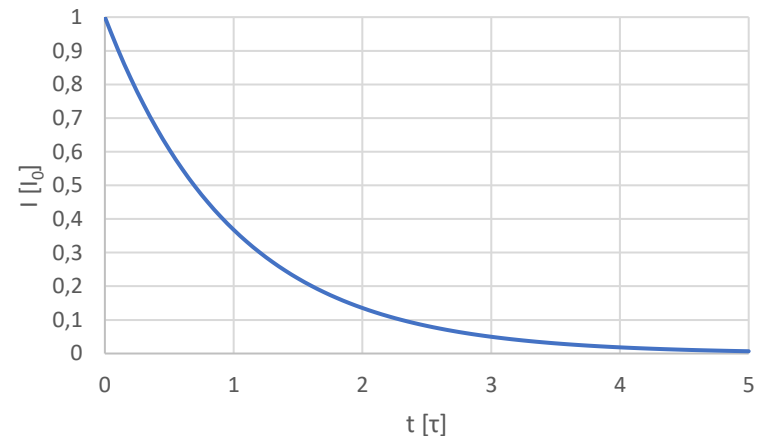
Capacitors (3)

- Charging (continued)
 - Solution of the equation
 - $Q = CU - CUe^{-\frac{1}{RC}t}$
 - $\tau = RC$ (time constant)
 - Course of the voltage
 - $U_C = \frac{Q}{C}$
 - $U_C = U(1 - e^{-\frac{t}{\tau}})$
 - Course of the current
 - $I = \dot{Q}$
 - $I = \frac{U}{R}e^{-\frac{t}{\tau}}$

Course of Voltage



Course of Current



Capacitors (4)

- Discharging

- Basic Relations

- $R = \frac{U}{I}$

- $I = \frac{dQ}{dt} = \dot{Q}$

- $Q = C \cdot U$

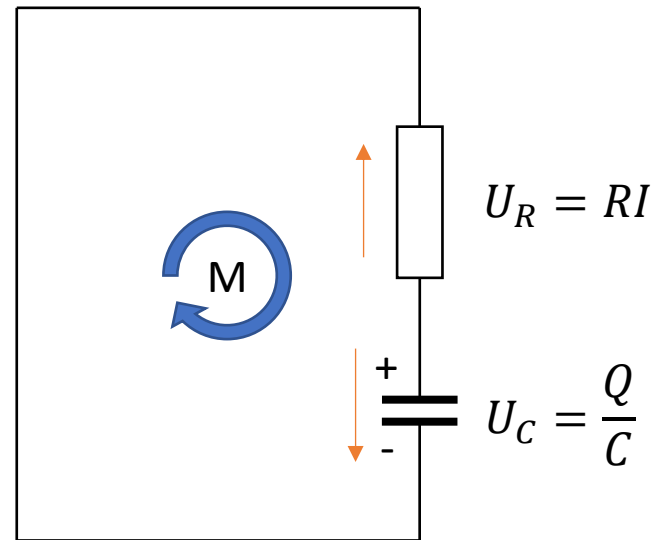
- Kirchhoff's circuit law

- $-U_R + U_C = 0$

- $-R\dot{Q} + \frac{Q}{C} = 0$

- Differential Equation

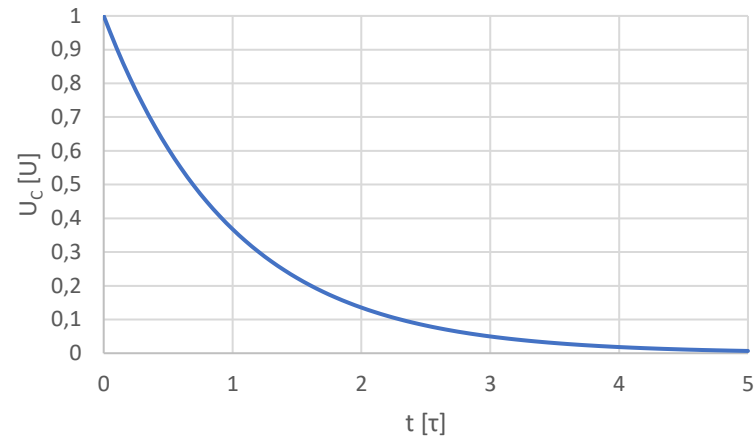
- $\dot{Q} = -\frac{1}{RC} Q$



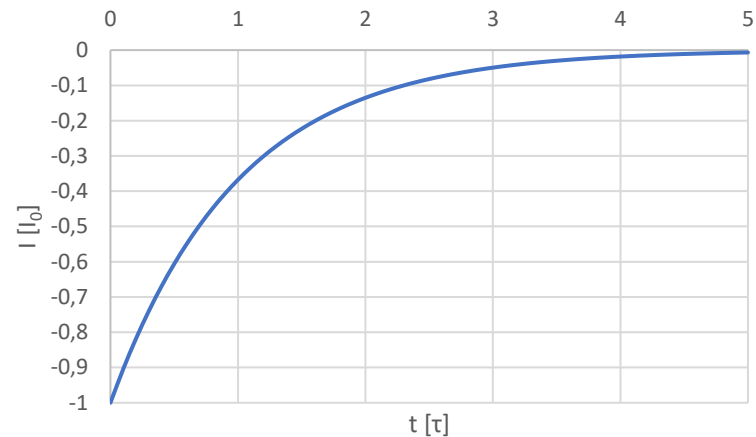
Capacitors (5)

- Discharging (continued)
 - Solution of the equation
 - $Q = CUe^{-\frac{1}{RC}t}$
 - $\tau = RC$ (time constant)
 - Course of the voltage
 - $U_C = \frac{Q}{C}$
 - $U_C = U(e^{-\frac{t}{\tau}})$
 - Course of the current
 - $I = \dot{Q}$
 - $I = -\frac{U}{R}e^{-\frac{t}{\tau}}$

Course of Voltage



Course of Current



Capacitors (6)

- Time Constant

- $\tau = RC$

- Half-value Time

- $e^{-\frac{t_h}{\tau}} = \frac{1}{2}$

- $t_h = \tau \ln 2$

- $t_h = 0.69\tau$

- Full Charge Time

- $t_f = 5\tau$

- $e^{-5} = 0.0067$

- $1 - e^{-5} = 0.993$

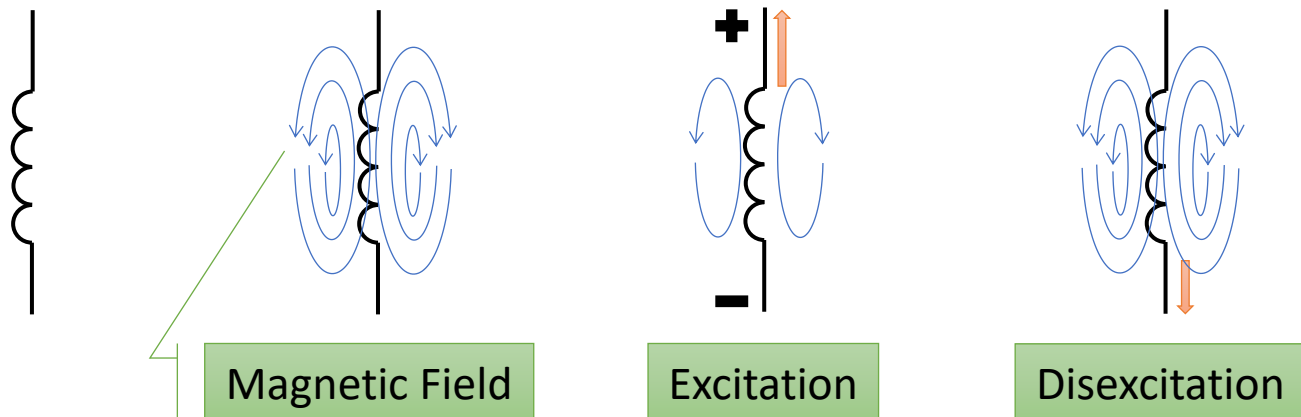
τ	0	1	2	3	4	5
Charge	0%	63%	86%	95%	98%	99%
Discharge	100%	37%	14%	5%	2%	1%

τ	0.0	0.1	0.2	0.3	0.4	0.5
Charge	0%	10%	18%	26%	33%	39%
Discharge	100%	90%	82%	74%	67%	61%

Inductors (1)

- Basic Features

- Stores electric energy when current flows through it
- The energy gets stored in a magnetic field
- A magnetic field can be excited and disexcited
- The induced current opposes the change of the field



Inductors (2)

- Excitation

- Basic Relations

- $R = \frac{U}{I}$

- $U_L = -L \frac{dI}{dt} = -L\dot{I}$

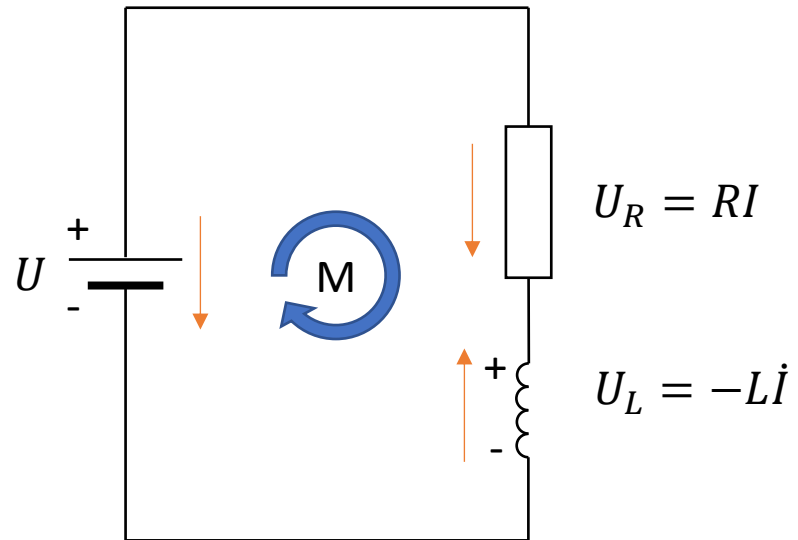
- Kirchhoff's circuit law

- $U_R - U_L - U = 0$

- $RI + L\dot{I} - U = 0$

- Differential Equation

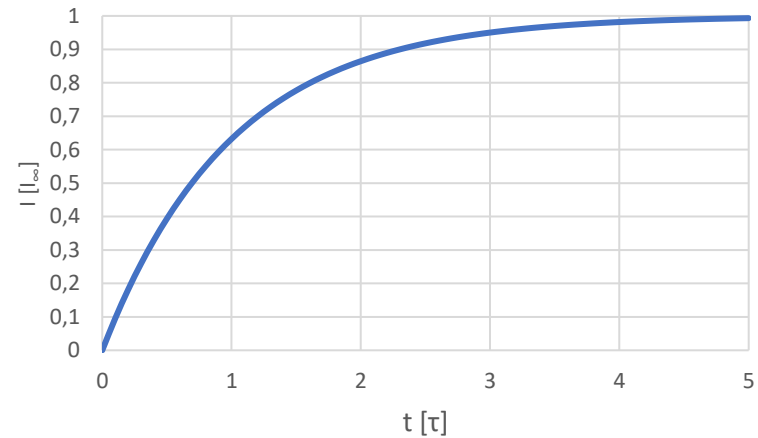
- $\dot{I} = \frac{1}{L}U - \frac{R}{L}I$



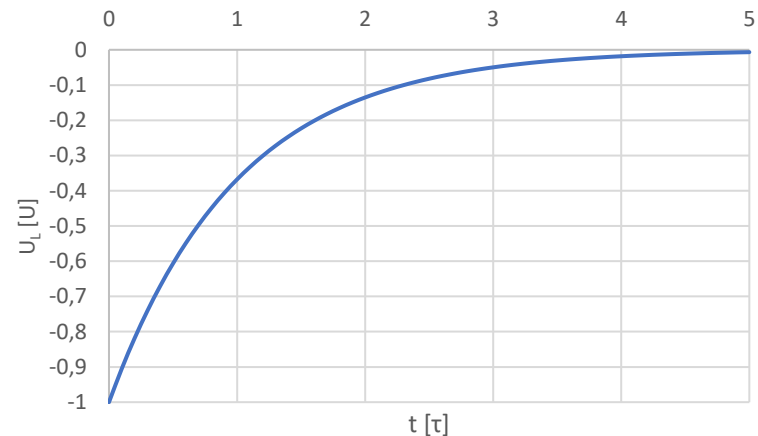
Inductors (3)

- Excitation (continued)
 - Solution of the equation
 - $I = \frac{U}{R} \left(1 - e^{-\frac{R}{L}t}\right)$
 - $\tau = \frac{L}{R}$ (time constant)
 - Course of the current
 - $I = \frac{U}{R} \left(1 - e^{-\frac{t}{\tau}}\right)$
 - Course of the voltage
 - $U_L = -L\dot{I}$
 - $U_L = -Ue^{-\frac{t}{\tau}}$

Course of Current



Course of Voltage



Inductors (4)

- Disexcitation

- Basic Relations

- $R = \frac{U}{I}$

- $U_L = -L \frac{dI}{dt} = -L\dot{I}$

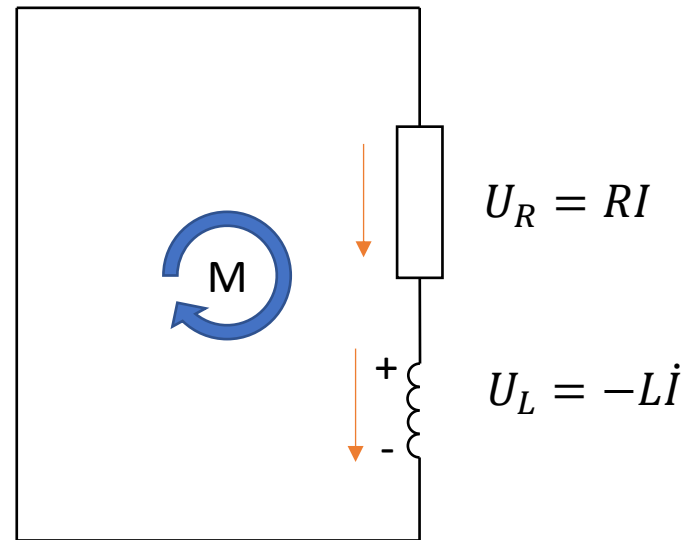
- Kirchhoff's circuit law

- $U_R + U_L = 0$

- $RI - L\dot{I} = 0$

- Differential Equation

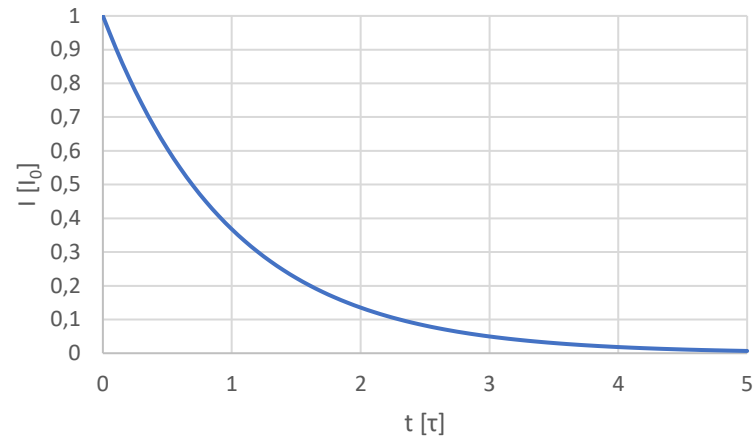
- $\dot{I} = \frac{R}{L}I$



Capacitors (5)

- Disexcitation (continued)
 - Solution of the equation
 - $I = \frac{U}{R} e^{-\frac{R}{L}t}$
 - $\tau = \frac{L}{R}$ (time constant)
 - Course of the current
 - $I = \frac{U}{R} e^{-\frac{t}{\tau}}$
 - Course of the voltage
 - $U_L = -L\dot{I}$
 - $U_L = U e^{-\frac{t}{\tau}}$

Course of Current



Course of Voltage

