

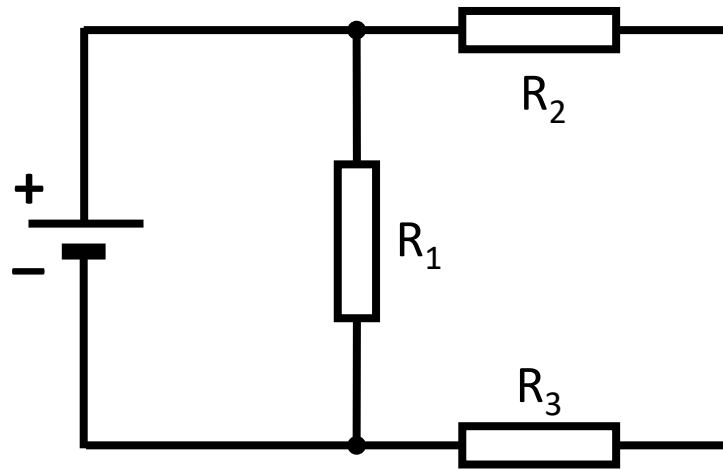
Kirchhoff's Circuit Laws

Electrical Engineering

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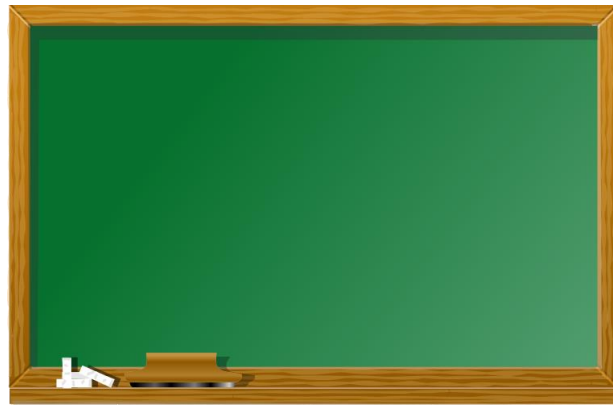
Kirchhoff's Circuit Laws (1)

- Complex Circuits
 - Are completely described by current and voltage
 - The junctions tell us something about current
 - The loops tell us something about voltage



Kirchhoff's Circuit Laws (2)

- Calculation of Unknown Circuit Quantities
 1. Make a list of all known values
 2. Mark all currents and voltages including arrows
 3. Write down the equations for all junctions and loops
 4. Set up the equations for all resistors
 5. Set up the linear system of equations and solve it



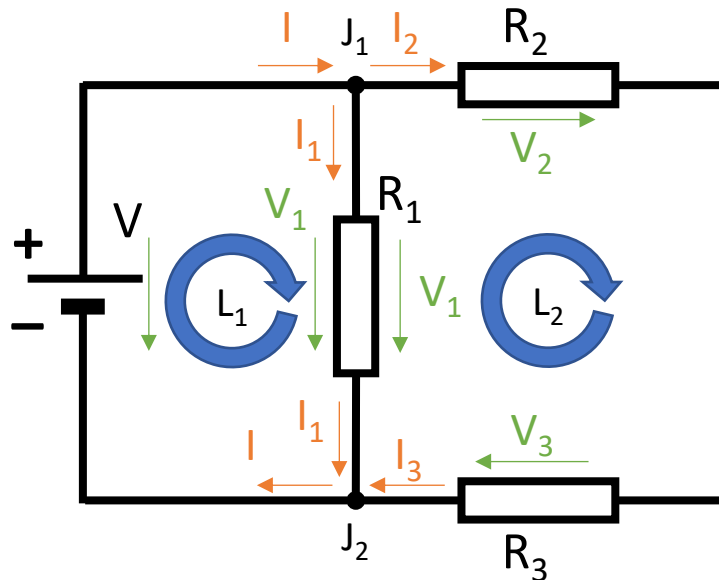
Kirchhoff's Circuit Laws (3)

- Example

- Make a list of all known values
- Mark all currents and voltages including arrows

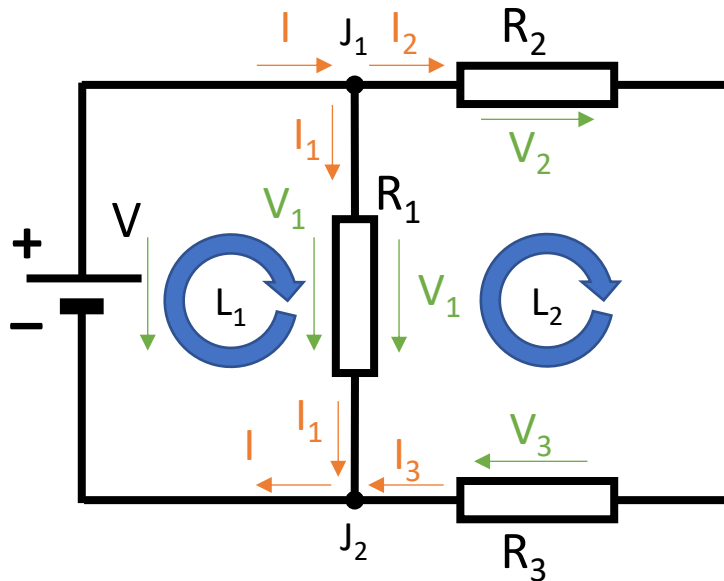
Known Values:

- $V = 5 \text{ V}$
- $R_1 = 200 \Omega$
- $R_2 = 100 \Omega$
- $R_3 = 50 \Omega$



Kirchhoff's Circuit Laws (4)

- Example (continued)
 - Write down the equations for all junctions and loops



Meshes:

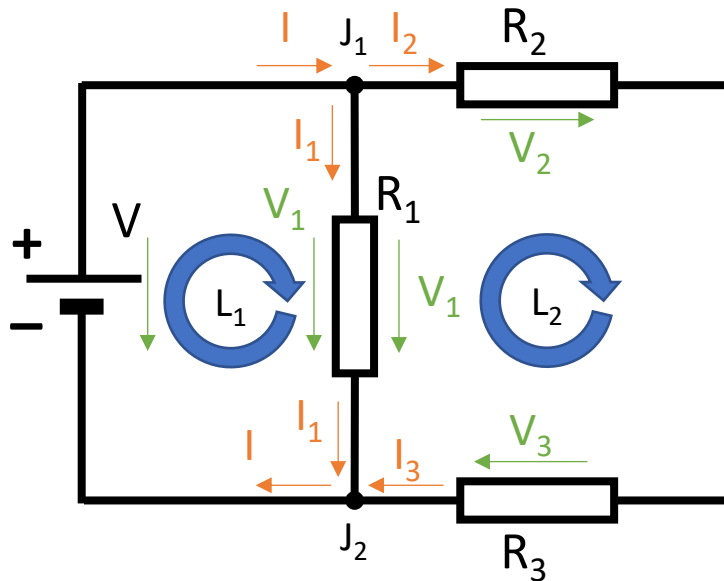
- $V_1 - V = 0$
- $V_1 + V_2 - V = 0$
- $V_2 + V_3 - V_1 = 0$

Junctions:

- $I - I_1 - I_2 = 0$
- $I_1 + I_3 - I = 0$

Kirchhoff's Circuit Laws (5)

- Example (continued)
 - Set up the equations for all resistors



Resistors:

- $V_1 = R_1 \cdot I_1$
- $V_2 = R_2 \cdot I_2$
- $V_3 = R_3 \cdot I_3$

Kirchhoff's Circuit Laws (6)

- Example (continued)
 - Set up the linear system of equations

Known Values:

- $V = 5 \text{ V}$
- $R_1 = 200 \ \Omega$
- $R_2 = 100 \ \Omega$
- $R_3 = 50 \ \Omega$

Meshes:

- $V_1 - V = 0 \quad \rightarrow \quad V_1 = 5$
- $V_1 + V_2 - V = 0 \quad \rightarrow \quad V_1 + V_2 = 5$
- $V_2 + V_3 - V_1 = 0 \quad \rightarrow \quad -V_1 + V_2 + V_3 = 0$

Junctions:

- $I - I_1 - I_2 = 0 \quad \rightarrow \quad J - J_1 - J_2 = 0$
- $I_1 + I_3 - I = 0 \quad \rightarrow \quad J_1 + J_3 - J = 0$

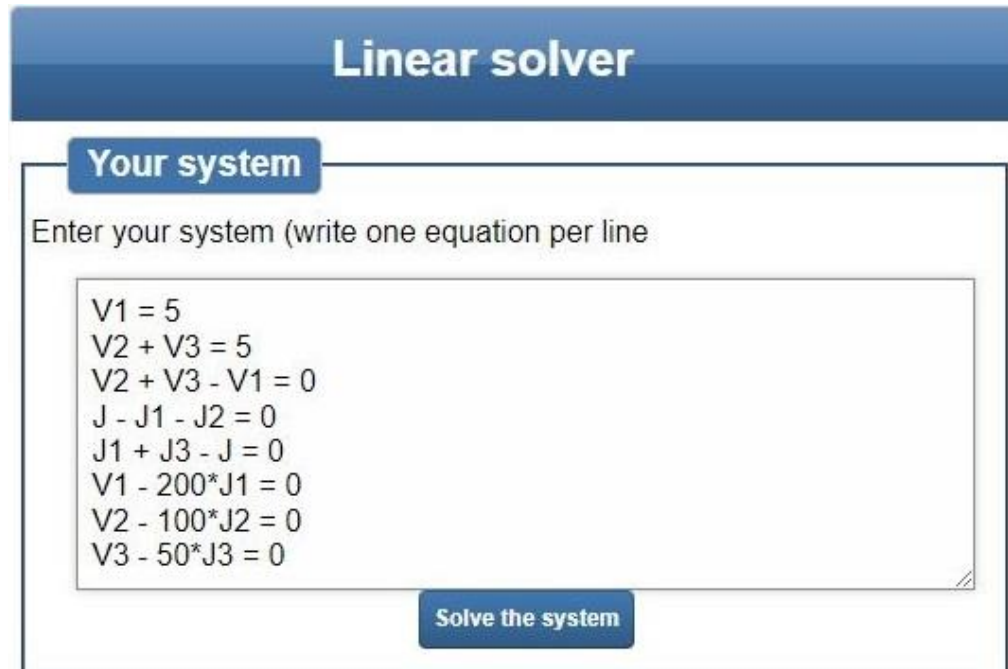
Resistors:

- $V_1 = R_1 \cdot I_1 \quad \rightarrow \quad V_1 - 200 J_1 = 0$
- $V_2 = R_2 \cdot I_2 \quad \rightarrow \quad V_2 - 100 J_2 = 0$
- $V_3 = R_3 \cdot I_3 \quad \rightarrow \quad V_3 - 50 J_3 = 0$

System of Equations

Kirchhoff's Circuit Laws (7)

- Example (continued)
 - Provide the equations to a linear solver



The image shows a web-based linear solver interface. At the top is a blue header with the text "Linear solver". Below this is a section titled "Your system" with a sub-header "Enter your system (write one equation per line)". A text area contains the following system of equations:

$$\begin{aligned}V_1 &= 5 \\V_2 + V_3 &= 5 \\V_2 + V_3 - V_1 &= 0 \\J - J_1 - J_2 &= 0 \\J_1 + J_3 - J &= 0 \\V_1 - 200 \cdot J_1 &= 0 \\V_2 - 100 \cdot J_2 &= 0 \\V_3 - 50 \cdot J_3 &= 0\end{aligned}$$

At the bottom of the text area is a blue button labeled "Solve the system".

Source: <https://wims.univ-cotedazur.fr/wims/wims.cgi?module=tool/linear/linsolver.en>

Kirchhoff's Circuit Laws (8)

- Example (continued)
 - Solve the linear system of equations

Linear solver

You have entered the system

Source: <https://wims.univ-cotedazur.fr/wims/wims.cgi?module=tool/linear/linsolver.en>

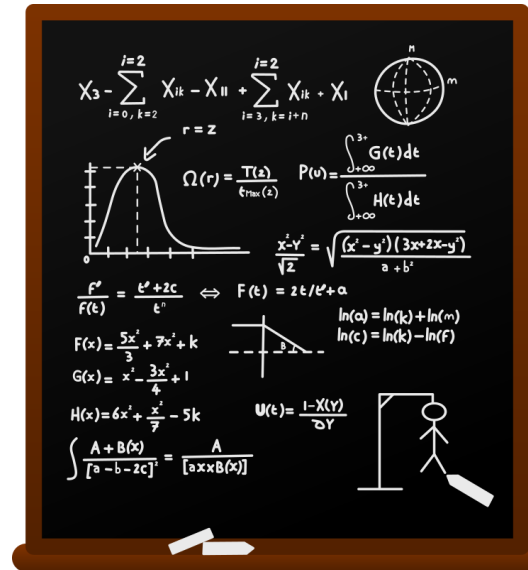
$$\left\{ \begin{array}{rcl}
 V_1 & & = 5 \\
 -V_1 + V_2 + V_3 & & = 5 \\
 J - J_1 - J_2 & & = 0 \\
 -J + J_1 + J_3 & & = 0 \\
 V_1 - 200J_1 & & = 0 \\
 V_2 - 100J_2 & & = 0 \\
 V_3 - 50J_3 & & = 0
 \end{array} \right.$$

i This system has a unique solution, which is: $j = 7/120$, $j_1 = 1/40$, $j_2 = 1/30$, $j_3 = 1/30$, $v_1 = 5$, $v_2 = 10/3$, $v_3 = 5/3$

Kirchhoff's Circuit Laws (9)

- Example (finished)

- $V = 5 \text{ V}$
- $V_1 = 5 \text{ V}$
- $V_2 = 3.33 \text{ V}$
- $V_3 = 1.67 \text{ V}$
- $I = 58.33 \text{ mA}$
- $I_1 = 25 \text{ mA}$
- $I_2 = 33.33 \text{ mA}$



All currents and voltages are determined